Supersonic Flow Past Pointed Bodies

J. C. South Jr.* and P. A. Newman†
NASA Langley Research Center, Hampton, Va.

The method of integral relations has been used to calculate supersonic flow past pointed bodies of revolution. Results of the second (two-strip) approximation are presented which further support the applicability of the method in purely supersonic flows. In particular, the question of an incorrect zone of influence is examined in detail. It is seen that, even though the surface boundary signals propagate along the normal to the surface rather than along the characteristic curve, the integral method partially compensates for this anomalous behavior. Results for the pressure gradient and shock-wave curvature at the tip of an ogive agree with exact calculations. Entire pressure distributions are shown for segmented shapes, which have discontinuous surface curvatures. These results compare well with those obtained by the method of characteristics, except in a case where the gradients normal to the body are severe.

Introduction

The method of integral relations is a numerical method for solving a system of partial differential equations by converting them to an approximating system of ordinary differential equations. The method and its applications in fluid mechanics have been reviewed recently by Belotserkovskii and Chushkin.[1] The obvious advantage is that highly efficient techniques for solving ordinary equations have been developed for high-speed digital computers. The major disadvantage of all integral methods is lack of detail.

Most applications of the method of integral relations have been in mixed-flow problems, where the main emphasis was placed on the solution in the subsonic-transonic flow region. Many investigators suggest using the numerical method of characteristics for continuation into the supersonic flow region. If great accuracy and detail are required and if the gas is ideal, the method of characteristics is certainly the superior technique. The inclusion of real-gas effects adds considerably to the computation time, however, and the consideration of three-dimensional flows introduces enormous complexity. Here, then, the greater speed and simplicity of the integral method become more attractive and deserve further consideration.

Xerikos and Anderson[2] questioned the applicability of the integral method in supersonic-flow regions for good reason: the influence of the surface boundary propagates along the normal coordinate rather than along the left-running characteristic. Traugott,[3,4] however, used the first (one-strip) approximation of the integral method far into the supersonic region of blunt-nosed shapes and obtained useful results. South[5,6] used the method in purely supersonic-flow problems, namely, nonequilibrium flow past wedges and cones with attached shock waves; the results for shock-wave shape and pressure distribution were in good agreement with those obtained by the method of characteristics. Furthermore, it was found that the integration step size had to be controlled by a stability criterion that was related to the characteristic curves. Thus, it appears that, in spite of an incorrect zone of influence, the approximating systems retain much of the character of the exact hyperbolic-type partial differential equations. In this note, further results are presented which support the applicability of the method in the supersonic region.

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* Aerospace Engineer. Member AIAA.
† Aerospace Engineer.

Method

We consider the two-strip integral method calculation of supersonic flow past pointed bodies of revolution as shown in Fig. 1. The governing equations are written in a body-oriented, curvilinear s, n coordinate system.[7] The shock layer is divided into two strips, and each of the three partial differential equations (continuity, and s and n momentum‡) is integrated across both strips using quadratic-interpolation polynomials for the integrands. The result is six ordinary differential equations that comprise the basic approximating system. They relate the s derivatives of the flow variables along the shock wave, the midline (dashed curve in Fig. 1), and the surface. The equations are similar to those in Appendix B of Ref. 6, specialized for an ideal (or equilibrium) gas, with the addition of terms arising because of surface curvature $K(s) = -d\theta/ds$.

In each of the six basic differential equations of the approximating system, the coefficients of the derivatives contain the factor $\delta(s)$. The shock attachment condition $\delta(0) = 0$ causes these coefficients to vanish. If we require a regular solution at $s = 0$, then the remaining algebraic terms in the differential equations must also vanish simultaneously at this

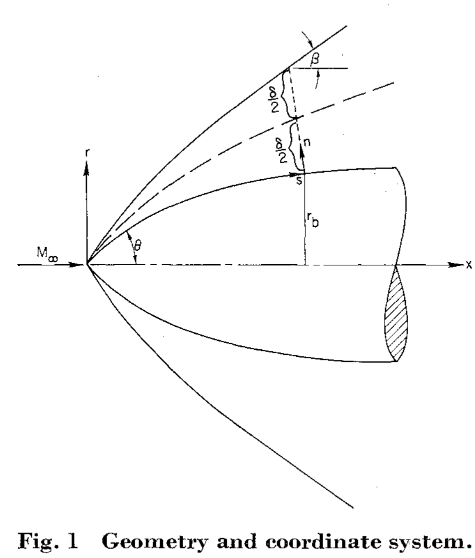

Fig. 1 Geometry and coordinate system.

‡ Previous ideal-gas studies[1] use the law of conservation of streamline entropy in lieu of the s-momentum equation. We prefer the present more general approach when it introduces no difficulty.

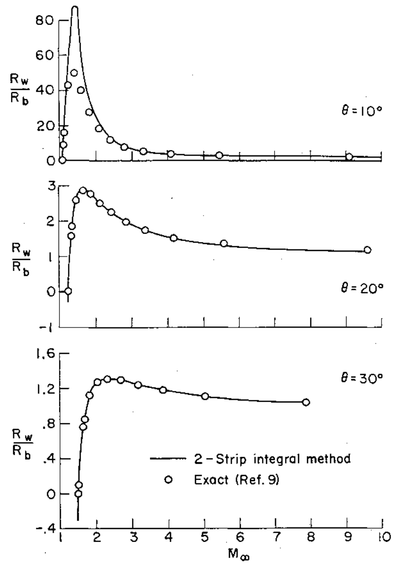

Fig. 2 Ratio of shock-wave curvature radius to body surface curvature radius freestream Mach number at the ogive tip.

point. This regularity condition yields six nonlinear algebraic equations that, coupled with the energy and state equations and the shock-wave relations, specify all the initial values for given freestream conditions and semivertex angle $\theta(0)$. This approximate conical-flow solution agrees very well with the exact Taylor-Maccoll solution for an ideal gas[6] and equilibrium air.[8]

The initial derivatives of the flow variables are in an indeterminate form and must be evaluated to start the numerical integration. Application of L'Hospital's rule gives a system of algebraic equations that are linear in the initial flow-variable derivatives. All of these derivatives are linearly dependent on the body tip curvature $K(0)$. Numerical integration of the system is straightforward if the integration step size is limited by the approximate hyperbolic stability criterion.

Numerical Results

In this note, results are presented for an ideal gas with $\gamma = 1.4$. The adaptation of the method to equilibrium-gas calculations is in progress. First, we illustrate the solution at the tip of an ogive of revolution. Figure 2 shows the ratio of shock-wave curvature radius to body surface curvature radius R_w/R_b as a function of M_∞. The exact numerical solutions were obtained in Refs. 9 and 10; our results are compared with those of Ref. 9, since they are more complete. For small semivertex angles ($\theta \lesssim 10°$) and moderate supersonic Mach numbers, the present method gives practically meaningless results for R_w/R_b. The reason for this is that the present method yields an approximate solution for $d\beta/ds$, which becomes very small in the range just described. Since $R_w/R_b \propto (d\beta/ds)^{-1}$, small errors in the present approximation are considerably magnified. All other flow-variable gradients given by the present method are one or more orders of magnitude larger than $d\beta/ds$ in this range and are well behaved.

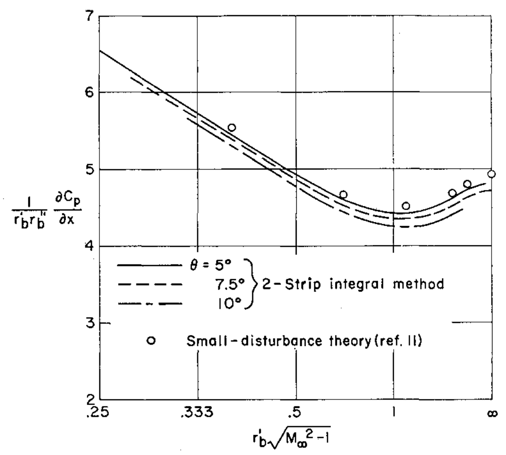

Fig. 3 Surface pressure coefficient gradient a combined supersonic-hypersonic similarity parameter at the ogive tip.

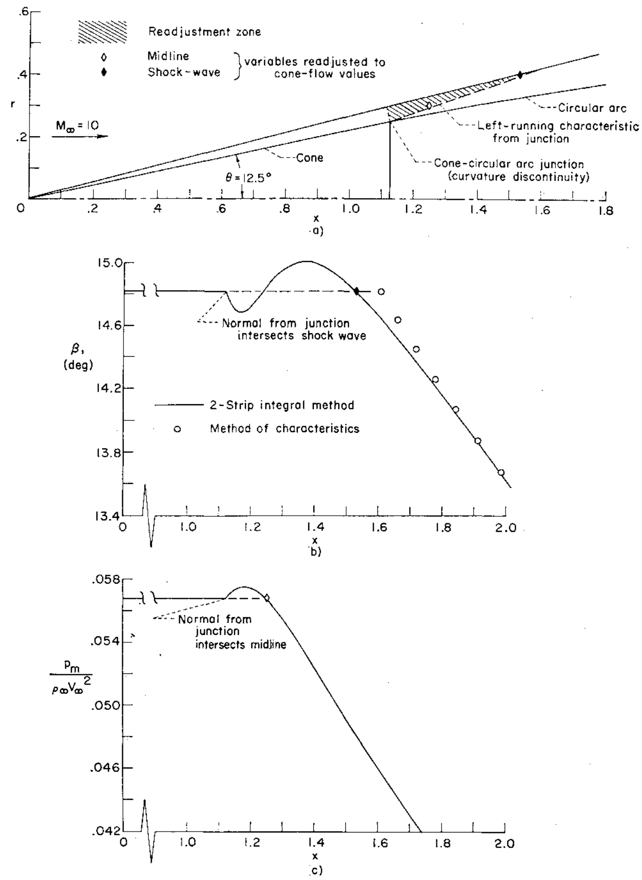

Fig. 4 Zone of influence of surface curvature discontinuity.

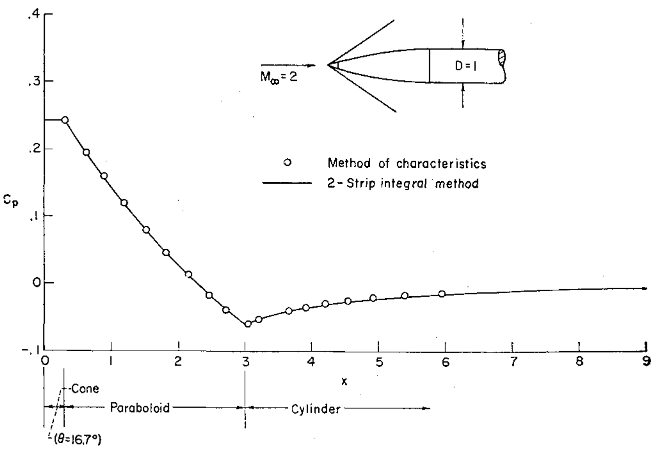

Fig. 5 Surface pressure coefficient axial distance for a cone-paraboloid-cylinder ($\theta = 16.7°$, $M_\infty = 2$).

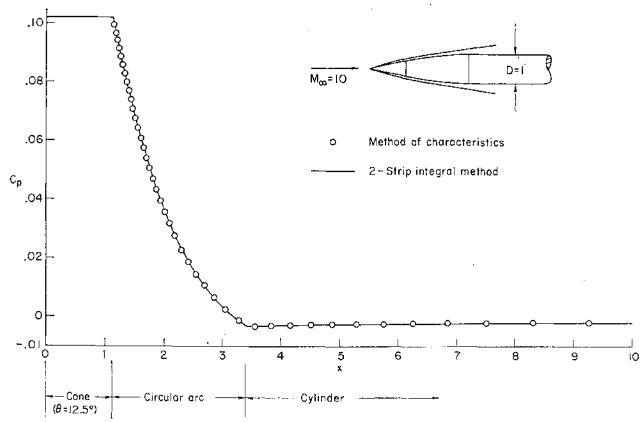

Fig. 6 Surface pressure coefficient axial distance for a cone-circular-arc-cylinder ($\theta = 12.5°$, $M_\infty = 10$).

The gradient of the surface pressure coefficient at the tip of an ogive is presented in Fig. 3 in the form suggested by Van Dyke's[11] hypersonic small-disturbance theory. The abscissa of Fig. 3 is the combined supersonic-hypersonic similarity parameter. As the semivertex angle θ decreases, the present results converge uniformly on those of the small-disturbance theory, which is exact in the limit $\theta \to 0$.

The result of a detailed examination of the zone of influence is pictured in Fig. 4. Figure 4a is a scale drawing of the flow at $M_\infty = 10$ past a cone followed by a circular arc segment. The left-running characteristic curve from the junction of the cone and circular-arc segment (where the surface curvature changes discontinuously from zero to a positive constant) is shown. Upstream of that curve, undisturbed conical flow should exist. In the present method, however, the curvature discontinuity is signaled immediately at all points along the normal n from the junction. The subsequent behavior of the variables amounts to a compensation, in part, for the spurious disturbance and a readjustment to the initial cone-flow values of the variables. This "readjustment zone" is the shaded region in Fig. 4a. The variation of the shock angle in this zone is shown in Fig. 4b, and the nondimensional midline pressure variation is shown in Fig. 4c. The diamond symbols indicate the point where the variables returned to their initial cone-flow values in Figs. 4b and 4c; the proximity of these points to the characteristic from the junction is seen in Fig. 4a. In Fig. 4b, the shock-angle variation following the readjustment zone is seen to agree well with the results obtained by the method of characteristics. The initial decrease of the shock angle allows for some correction of the shock-wave ordinate.

Figures 5–7 give the surface pressure coefficient as a function of axial distance for three different cases. Figure 5 presents results for a cone-paraboloid-cylinder with a semivertex angle of 16.7° at $M_\infty = 2$. Figure 6 depicts a cone-circular-arc-cylinder with a semivertex angle of 12.5° at $M_\infty = 10$. (This is the same case as is shown in Fig. 4.) Note that the agreement with characteristics is excellent in both cases. In Fig. 7, results are given for a cone-circular-arc-cylinder with semivertex

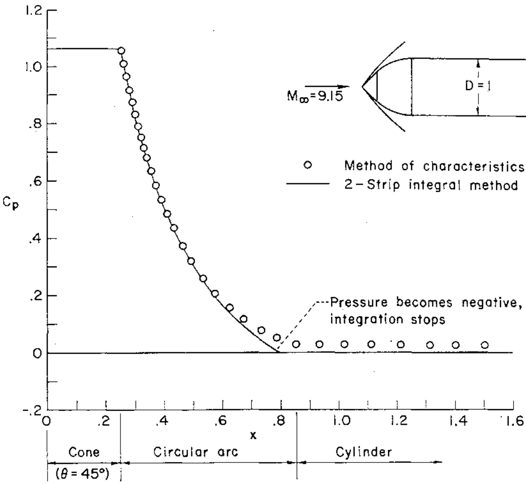

Fig. 7 Surface pressure coefficient axial distance for a cone-circular-arc-cylinder ($\theta = 45°$, $M_\infty = 9.15$).

angle of 45° at $M_\infty = 9.15$. It can be seen that the integration stopped when the pressure (not C_p) became negative. It is felt that this is due to the strong entropy gradients that are present in the shoulder region and not related to the manner in which the surface boundary signals are propagated. Apparently the accuracy of the two-strip method is not adequate in this case.

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